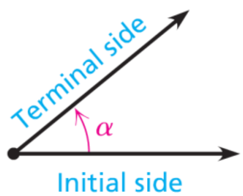
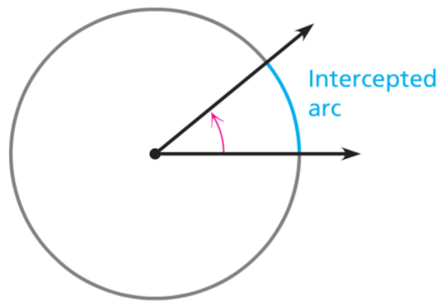
Degree Measure of Angles

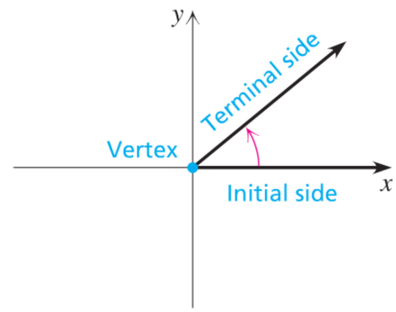
An **angle** can be formed by rotating one ray away from a fixed ray indicated by an arrow. The fixed ray is the **initial side** and the rotated ray is the **terminal side**. An angle whose vertex is the center of a circle is a **central angle**, and the arc of the circle through which the terminal side moves is the **intercepted arc**. An angle in **standard position** is located in a rectangular coordinate system with the vertex at the origin and the initial side on the positive *x*-axis.



Angle α



Central angle



Angle in standard position

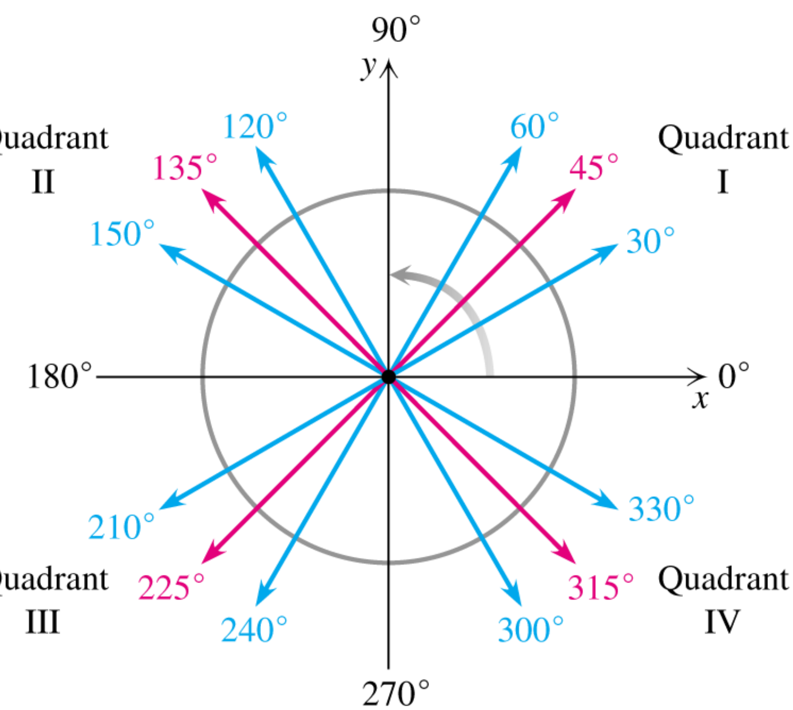
Degree Measure of Angles

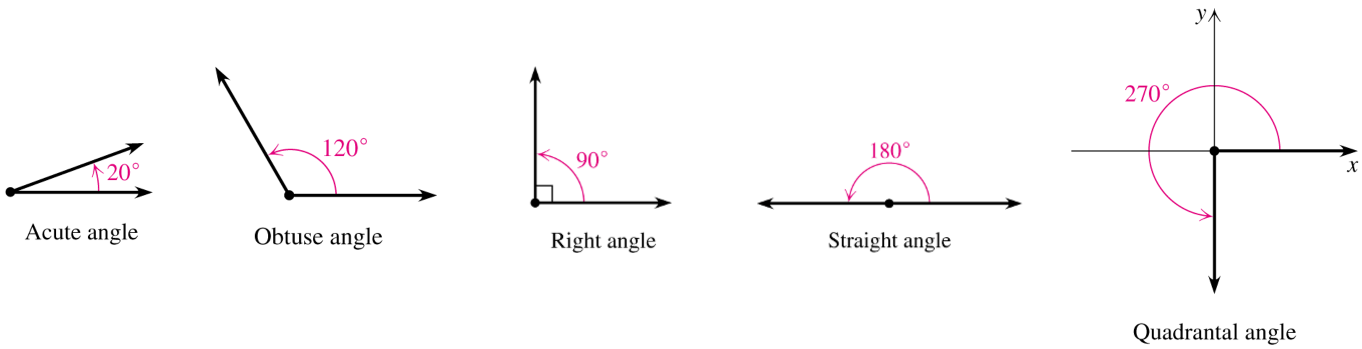
The measure, $m(\alpha)$, of an angle α is the amount of rotation from the initial side to the terminal side, and is found by using any circle centered at the vertex. An angle that forms a complete circle arc is 360°.

The **degree measure of an angle** is the number of degrees in the intercepted arc of a circle centered at the vertex.

Counterclockwise rotation—positive angle **Clockwise rotation—negative angle**

An angle in standard position is said to lie in the quadrant where its terminal side lies.





Acute angle—An angle with a measure between 0° and 90°.
Obtuse angle—An angle with a measure between 90° and 180°.
Straight angle—An angle with a measure of exactly 180°.
Right angle—An angle with a measure of exactly 90°.
Quadrantal angle—An angle in standard position whose terminal side is on an axis.

The terminal side of an angle may be rotated in either a positive or negative direction to get to its final position. It can also be rotated for more than one revolution in either direction. Two angles with the same terminal side are called **coterminal angles.**

Coterminal Angles—Angles α and β are coterminal if and only if there is an integer k such that $m(\beta) = m(\alpha) + k360°$. To find coterminal angles in degrees, add and subtract multiples of 360°.

Examples: Find two positive angles and two negative angles that are coterminal with each angle:
a) 23° b) –146°

Examples: Determine whether the angles in each pair are coterminal:
a) –128° and 592° b) 8° and –368°

Example: Draw each angle in standard position, then name the quadrant in which the terminal side lies.
a) 255° b) –650° c) 1360°

Minutes and Seconds

Historically, angles were measured by using the **degrees-minutes-seconds (DMS) format**, but with calculators it is convenient to have some fractional parts of degrees written in decimal form. Each degree is divided into 60 equal parts called **minutes** (n'), and each minute is divided into 60 equal parts called **seconds** (n'').

1 degree = 60 minutes   1 minute = 60 seconds   1 degree = 3600 seconds

Examples: Convert each angle to decimal degrees. When necessary, round to four decimal places.

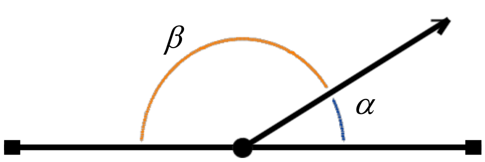
a) $18°24'$ b) $-10°15'42''$ c) $27°10'20''$

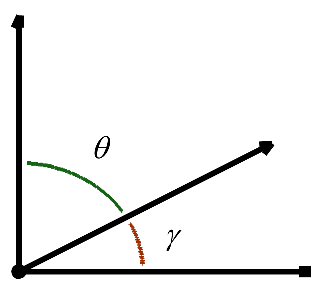
Examples: Convert each angle to degree-minutes-seconds format. Round to the nearest whole number of seconds.

a) $56.6°$ b) $-17.45°$ c) $28.348°$

Supplementary and Complementary Angles

Recall that two acute angles are called **complementary angles** if their measures add to 90°. Two angles, either a pair of right angles or one acute angle and one obtuse angle, are called **supplementary angles** if their measures add to 180°. In the diagrams below, the angles α and β are supplementary angles while the pair γ and θ are complementary angles.





Examples: Find a supplementary and complementary angle for each given angle.

a) 111.371° b) 37 28'17"